5 Deformation, Storage, and General Flow Equations

5.1 Introduction

Changes in head and pore water pressure cause deformation of the solid matrix that holds the water, deformation that has a range of impacts including subsidence, fissures, liquefaction, slope failure, and faulting. Pore pressure changes and matrix deformation are also key aspects of transient (time-dependent) groundwater flow. The last sections of this chapter introduce the general equations of transient groundwater flow, which follow from Darcy's law, mass balance, and storage concepts. The general equations of flow are the basis for mathematical models of groundwater flow, which are the subject of subsequent chapters.

5.2 Effective Stress

When the soil or rock matrix compresses or expands, it does so in response to changes in something called *effective stress*. To illustrate what this is, consider the vertical column shown in Figure 5.1. What is holding this column up? A reasonable assumption is that there is no net vertical supporting force on the sides of the column; this column does not support or drag down neighboring columns. The total weight of the column is borne by its base.

The weight of the column divided by the area of its base is called the total vertical stress and is given the symbol σ_{vt}. The units of stress are force/area, just like the units of pressure (N/m^2 or lb/ft^2, for example). In a rock or soil with a uniform wet density ρ, the total weight of the column at a depth b below the ground surface would be

$$W = \rho g b A \tag{5.1}$$

where g is gravitational acceleration and A is the cross-sectional area of the column. The total vertical stress at depth b is the weight per area

$$\sigma_{vt} = \rho g b \tag{5.2}$$

If the subsurface profile consists of n layers, each with a unique density, the total vertical stress is given by

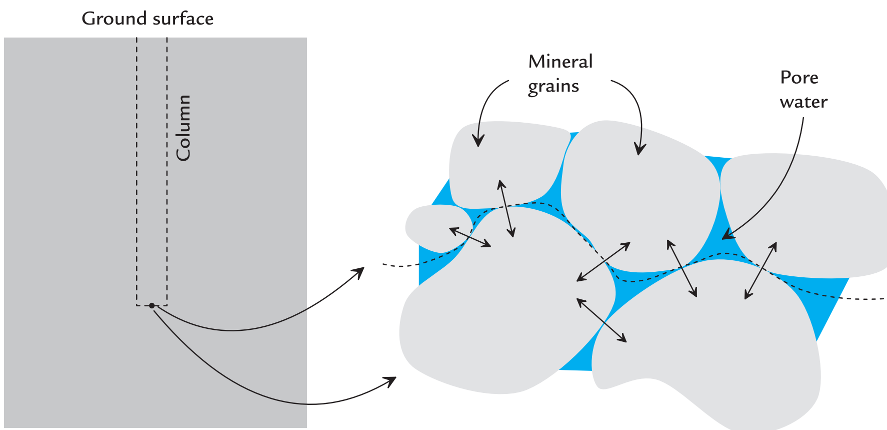

Figure 5.1 Column of soil supported by its base (left) and a close-up of the soil at the base (right). Two things support the column: forces across grain-to-grain contacts (effective stress) and pressure in the pore water.

$$\sigma_{vt} = g \sum_{i=1}^{n} \rho_i b_i \tag{5.3}$$

where ρ_i and b_i are the density and thickness of the ith layer. The vertical force per area at the base is σ_{vt}.

Imagine drawing a roughly planar surface through the interconnected pore spaces at the base, as shown by the dotted line in the right side of Figure 5.1. Looking closely at this surface, we see that the total stress is borne by two types of forces that act across the surface. One is the force of the pore water pressure, and the other is the force in the solid matrix. The force of the pore water pressure P acts all across this surface. The force in the matrix acts through the network of grains in soils and in the matrix of rock. The matrix forces acting across the base divided by the area of the base is the vertical effective stress, σ_{ve}. The vertical effective stress plus the pore water pressure equals the total vertical stress:

$$\sigma_{vt} = P + \sigma_{ve} \tag{5.4}$$

The column is held up by two forces: the pore water pressure, and the matrix forces (effective stress).

In general, **effective stress** is the force/area acting through the solid matrix. The concept of effective stress was first described by Karl Terzaghi (1925), and it is a key concept in modern soil mechanics (see Terzaghi *et al.*, 1996, or Lambe and Whitman, 1979).

Example 5.1 Consider a sand that is unsaturated from the ground surface down to a depth of 4.5 ft, and saturated below that. The total unit weight of the unsaturated sand is $\rho g = 112$ lb/ft^3 and the total unit weight of the saturated sand is $\rho g = 125$ lb/ft^3. Assuming that the distribution of pore water pressures is hydrostatic, calculate σ_{vt}, P, and σ_{ve} at a depth of 12 ft.

The total stress is given by Eq. 5.3, summing the contributions of the unsaturated and saturated zones:

$$\begin{aligned}\sigma_{vt} &= (112\ \text{lb/ft}^3)(4.5\ \text{ft}) + (125\ \text{lb/ft}^3)(7.5\ \text{ft}) \\ &= 1442\ \text{lb/ft}^2\end{aligned}$$