Modeling Steady Flow with Basic Methods

6

6.1 Introduction

All of the methods presented in this chapter are for flow that is conceptualized as steady state, an assumption typically invoked to model long-term average flow conditions. The methods covered are all one- or two-dimensional, where the resistance to flow in one or two dimensions is neglected. These methods can be done with hand calculations or with the help of relatively simple computer programs. Where there is limited data about a site or where the problem geometry is not too complex, these simple analyses are often the best approach.

6.2 Aquifers with Uniform Transmissivity

For many flow problems in confined aquifers and some in unconfined aquifers, it is reasonable to construct a model that approximates the real system in the following ways:

1. The flow is steady state.
2. The resistance to vertical flow is neglected; only the resistance to horizontal flow is accounted for.
3. The aquifer transmissivity T is homogeneous and constant.

The general equations that govern flow with these assumptions are Eqs. 5.64 and 5.65. Several simple and useful solutions to these general equations are presented in this section.

6.2.1 Solution for Uniform Flow

One solution of the Laplace equation (Eq. 5.65) represents uniform flow in one direction, where the hydraulic gradient is constant over the whole x, y plane and the potentiometric surface is planar. On a large scale, the potentiometric surface of an aquifer is usually not planar. But if the area of interest is just a small portion of an aquifer, the head distribution within that area may be nearly planar and this solution can be useful.

This solution can be derived by observing that one possible set of solutions for the Laplace equation would have both

$$\frac{\partial^2 h}{\partial x^2} = 0 \qquad \text{and} \qquad \frac{\partial^2 h}{\partial y^2} = 0 \tag{6.1}$$

If the above equations are true, then integration of the above gives

$$\frac{\partial h}{\partial x} = A \qquad \text{and} \qquad \frac{\partial h}{\partial y} = B \tag{6.2}$$

where A and B are constants. Integrating both of these equations results in a solution of the form

$$h = Ax + By + C \tag{6.3}$$

where A, B, and C are constants. This solution represents uniform horizontal flow with a planar potentiometric surface. The constants A and B are the hydraulic gradients in the x and y directions, as Eq. 6.2 shows. The constant C moves the head surface up and down to different elevations without affecting the gradient. By itself, this solution represents flow in a uniform direction with a uniform hydraulic gradient everywhere in the x, y plane. If $A = B = 0$, this solution reduces to $h = C$, a stagnant condition with no gradient and no flow. Three points of known head are required to uniquely define the surface with the constants A, B, and C.

Example 6.1 Figure 6.1 shows a plan view of three observation (nonpumping) wells in a confined aquifer, with the heads measured at each. Determine the mathematical model for uniform flow that fits these observations. Use this model to predict the head at point P.

The model for uniform flow is Eq. 6.3. The three constants A, B, and C are unknown and there are three conditions, namely the heads at the three observation wells, that allow determination of the three unknowns. Equation 6.3, written for location M gives

$$\begin{aligned} h_M &= A(0) + B(0) + C \\ h_M &= C = 120.0 \end{aligned}$$

Equation 6.3 at location N gives

$$\begin{aligned} h_N &= A(500) + B(0) + C \\ h_N &= A(500) + 120.0 \end{aligned}$$

Solving this last equation for A results in $A = -4/500$. Equation 6.3 at location O gives

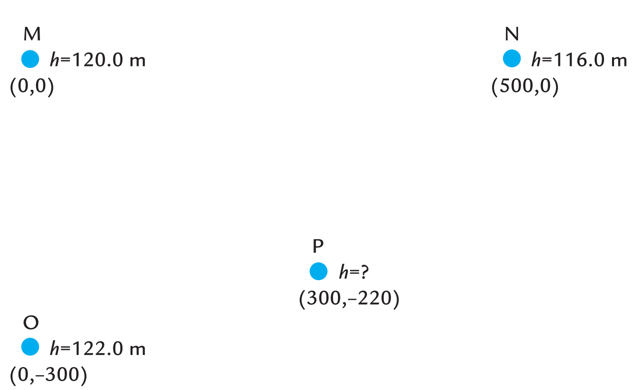

Figure 6.1 Plan view of three wells in a confined aquifer (Example 6.1). The wells are at points M, N, and O. The coordinates are listed below each well in the form (x, y).