7 Modeling Transient Flow with Basic Methods

7.1 Introduction

So far we have only examined models of steady-state groundwater flow, where discharges and heads do not change with time. This is reasonable when long-term average flows are considered, but unreasonable in many other situations. For example, transient flow is important when pumping wells start up or shut down, and with natural transients like drought and storms. Many transient analyses are so complicated that they must be carried out with the aid of computer programs. Some situations involving radial flow to wells are simple enough that they can be analyzed with hand calculations based on analytic solutions. The radial flow solutions are quite useful for analysis of pumping tests and for predicting drawdown near pumping wells.

7.2 Radial Flow in Aquifers with Uniform Transmissivity

7.2.1 The Theis Nonleaky Aquifer Solution

The Theis (1935) solution is commonly applied to analyze problems involving transient flow to a well. It is a solution to the general flow equation for transient two-dimensional horizontal flow with homogeneous, isotropic K (Eq. 5.63). The Theis solution assumes radial flow to a well of constant discharge in an infinite aquifer. Theis derived this solution by using research in the field of heat flow and noting the direct analogy between heat flow emanating from a long, straight wire and groundwater flow to a well. The geometry of the problem solved by Theis is illustrated in Figure 7.1.

We will not delve into how the Theis solution or other solutions presented in this chapter were derived. Suffice it to say that Laplace transforms are employed and the mathematics involved are beyond the scope of this book. Using the principle of superposition, Theis's solution can be added to or subtracted from any solution of the steady-flow general equation h_0, and the combined solution h will be a solution of the transient general equation, Eq. 5.63:

$$h = h_0(x, y) - \frac{Q}{4\pi T} W(u) \tag{7.1}$$

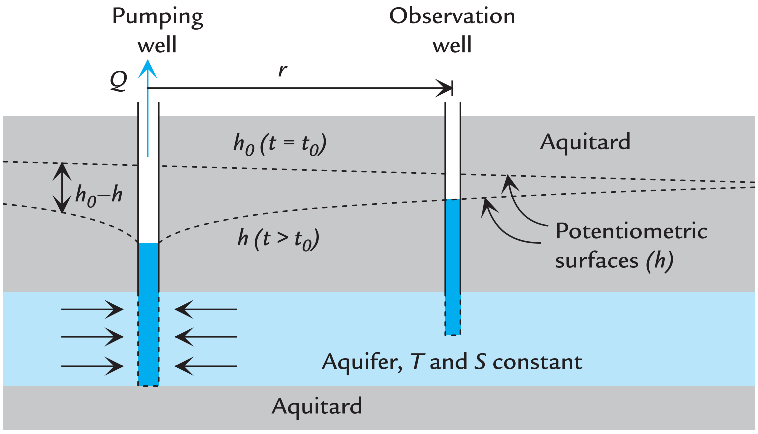

Figure 7.1 Vertical cross-section of transient radial flow to a well. The head before pumping starts is some steady-state distribution of heads, $h_0(x, y)$. The drawdown after pumping starts is radially symmetric about the pumping well, $[h_0 - h](r)$. For the solutions presented in this chapter, the aquifer T and S (or S_y) are assumed constant.

where $W(u)$ is known as the well function and u is a dimensionless parameter defined as

$$u = \frac{r^2 S}{4T(t - t_0)} \tag{7.2}$$

More typically, the Theis solution is written in terms of the drawdown $(h_0 - h)$ induced by the pumping well:

$$h_0 - h = \frac{Q}{4\pi T} W(u) \tag{7.3}$$

The well function is what mathematicians call the exponential integral E_1, which is written as

$$\begin{aligned} W(u) &= E_1(u) \\ &= \int_u^\infty \frac{e^{-m}dm}{m} \qquad (u < \pi) \end{aligned} \tag{7.4}$$

There is no closed-form expression for the exponential integral, but it can be closely approximated using a truncated series expansion as follows (Abramowitz and Stegun, 1972):

$$E_1(u) = -\gamma - \ln u + u - \frac{u^2}{2(2!)} + \frac{u^3}{3(3!)} - \frac{u^4}{4(4!)} + \cdots \tag{7.5}$$

where $\gamma = 0.5772157\ldots$ is Euler's constant. This function is tabulated in many mathematical handbooks. The curve in Figure 7.2 plots $W(u)$ vs. $1/u$ and is known as the Theis or nonequilibrium curve.

The simplifying assumptions of the Theis solution are listed below.

1. The aquifer is infinite in extent, with no constant head boundaries, no-flow boundaries, or any other heterogeneity.
2. The aquifer is homogeneous, with constant T and S over its infinite extent.
3. The well does not induce additional leakage or recharge through the top and bottom of the aquifer.
4. The well fully penetrates the aquifer, and there is only resistance to horizontal flow.