# Computer-Assisted Flow Modeling

# 8

## 8.1 Introduction

As discussed in Section 5.10, the modeling process begins with developing a conceptual model that is simpler than reality, then a mathematical model is constructed to simulate the conceptual model. If the conceptual model is simple enough, the corresponding mathematical model may be solved using hand calculations like those introduced in the preceding chapters. With more complex conceptual systems, more complex mathematical models are required. For such problems, the only practical approach is to use a computer program to do the computations for you.

Before computers were commonplace and inexpensive, complex groundwater flow problems were modeled using physical models or analogs. The physical model would be a miniature scaled model of the flow domain in a tank. The most common analog method uses the flow of electricity through a network of wires and resistors, where voltage is analogous to head, resistance is analogous to $1/K$ or $1/T$, current is analogous to discharge, and capacitance is analogous to storage. Computer simulation methods have essentially replaced physical and analog modeling, so these methods are not discussed in any further detail. The interested reader will find more detailed coverage of analog methods in Walton (1970) and other textbooks of that vintage.

At present, most computer programs for groundwater flow modeling are based on one of three methods: finite differences, finite elements, or analytic elements. A fourth less common method, the boundary integral equation method (BIEM), is conceptually similar to the analytic element method, but it is not discussed here. Interested readers are referred to Liggett and Liu (1983) for more on the BIEM. Each method has its particular strengths and weaknesses and no one method is the right tool for every problem. The following sections outline the essentials of each method.

## 8.2 Finite Difference Method

The finite difference method (FDM) is a numerical method that is quite versatile, relatively simple, and currently the most widely used method for flow modeling. The current standard FDM program is MODFLOW, a FORTRAN program developed by the

U.S. Geological Survey for three-dimensional flow modeling (McDonald and Harbaugh, 1988).

MODFLOW gained broad acceptance in the 1980s because it is versatile, well-tested, well-documented, and in the public domain. The formatted text input and output files associated with MODFLOW are cumbersome, but graphical user interface software shields the user from these details and makes using MODFLOW easy.

MODFLOW was programmed in a modular way, so that additional capabilities could be added over time. Some of the more important additions include variable-density flow (Sanford and Konikow, 1985), MODPATH for tracing pathlines (Pollock, 1989), an improved matrix solution procedure (Hill, 1990), better handling of water table boundary conditions (McDonald *et al.*, 1991), parameter estimation capability (Hill, 1992), and thin barriers to flow (Hsieh and Freckleton, 1993).

The following discussion of the finite difference method is only introductory. More detailed coverage of the method may be found in the MODFLOW manual (McDonald and Harbaugh, 1988) or texts by Anderson and Woessner (1992), Bear and Verruijt (1987), and Wang and Anderson (1982).

Instead of using analytic solutions to the general flow equations, the **finite difference method** uses a series of algebraic equations which are based on conservation of mass and Darcy's law. These algebraic equations are solved for unknown heads at discrete nodes located within an orthogonal network of nodes. One FDM scheme places nodes at the center of each block and another places them at the corners. MODFLOW uses block-centered nodes, so that is the scheme described here.

The modeled domain is subdivided or **discretized** into a grid of rectangular blocks or cells as shown in Figure 8.1. Within each block, the physical properties of the domain ($K_x, K_y, K_z, n$, and $S$) are assumed to be homogeneous. The domain can be made heterogeneous by assigning different properties to different blocks. It is assumed that the principal directions of hydraulic conductivity (see Section 3.2.2) line up with the orthogonal directions of the grid.

The dimensions of blocks vary, with smaller blocks in areas where more accuracy is required. The spacing of the grid boundaries in the $x$ and $y$ directions is fixed throughout the grid so that there are bands of narrower blocks that traverse the whole grid as shown in Figure 8.1. The horizontal boundaries that form the tops and bottoms of blocks can be staggered to form irregular surfaces so that model layers correspond to stratigraphic layers, as shown in Figure 8.2.

In transient FDM models, time is also discretized. Time is assumed to proceed in discrete steps, with an approximate solution determined at each time step. Continuity of flow at each node at each time step is based on storage changes from one time step to the next. The nuts and bolts of the FDM computations are summarized in the next section.

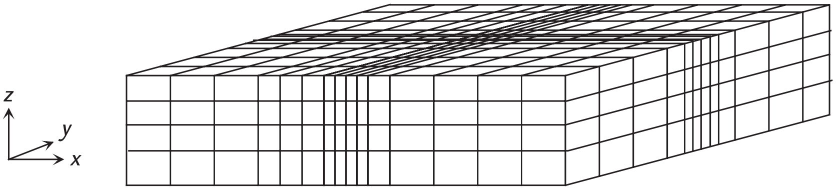

**Figure 8.1** Finite difference discretization of the domain.